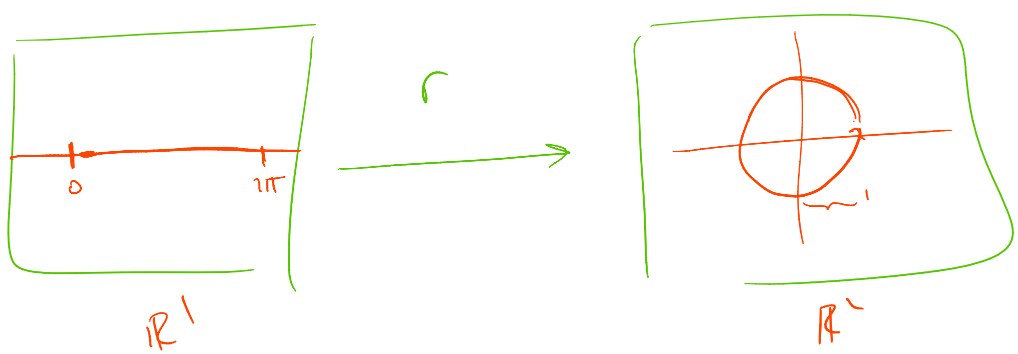
16.5 (divergence and curl)
16.6 (parametric surfaces and their areas)

Given circle of radius 1 @ origin

$\langle \cos(t), \sin(t) \rangle$ , $\langle \cos(7t^2), \sin(7t^2) \rangle$




**Def:** a parametric surface is a function $r: R^2 \to R^3$, $r(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

**EXAMPLE 2** Use a computer algebra system to graph the surface

$\mathbf{r}(u, v) = \langle (2 + \sin v) \cos u, (2 + \sin v) \sin u, u + \cos v \rangle$

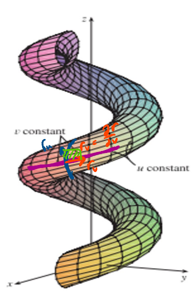



**FIGURE 5**

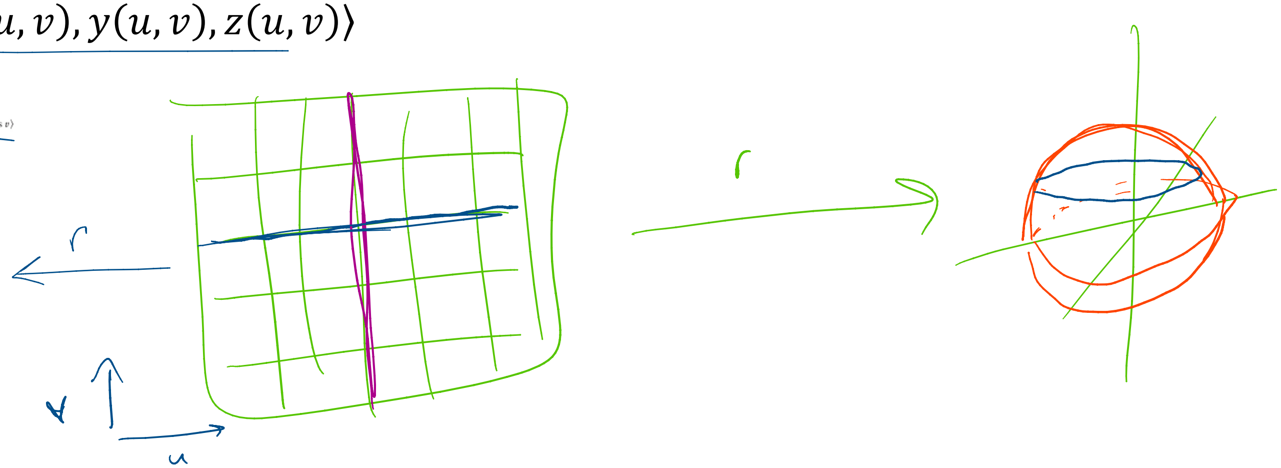
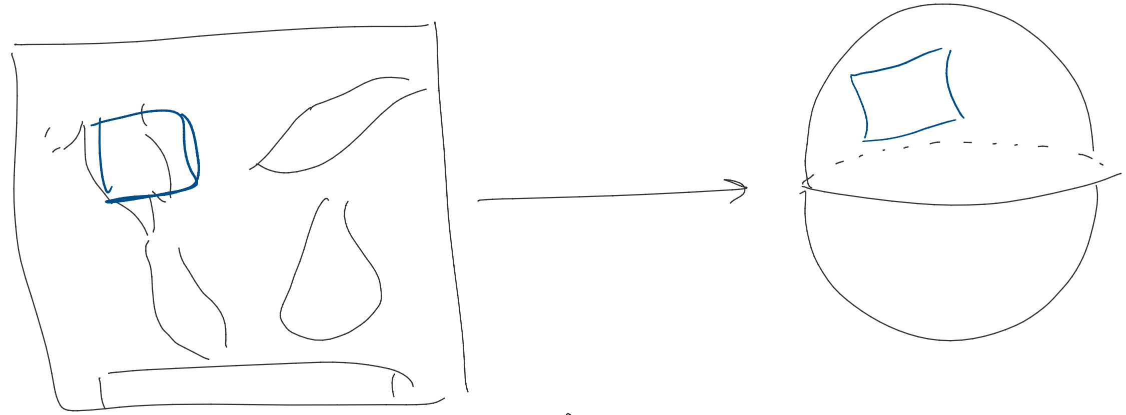


FIGURE 5

By computing $\frac{\partial r}{\partial u}$ and $\frac{\partial r}{\partial v}$ we can get formulae for the tangent plane and surface area.



param. the sphere

$r = (R\sin\varphi\cos\theta,\ R\sin\varphi\sin\theta,\ R\cos\varphi)$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$

$r(u_0, u_0)$

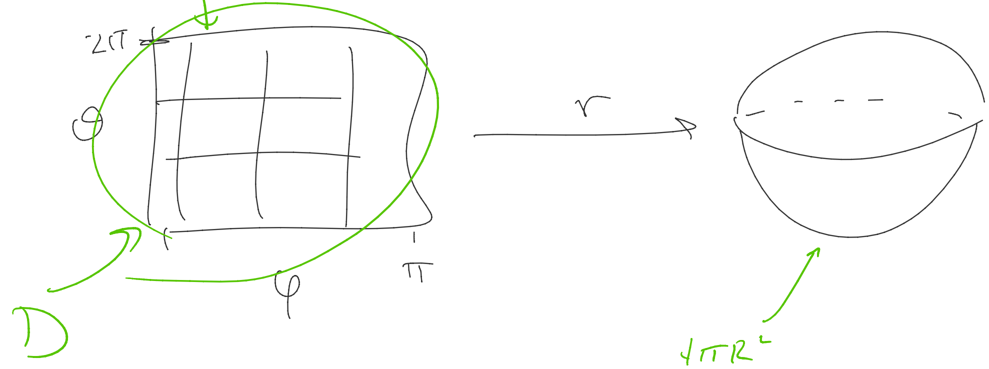
$\parallel$

**Tangent plane of $r$ at $r_0$:** normal $= r_u \times r_v$

**Surface area of $r$** $\iint_D dA =$

$dA = \|r_u \times r_v\|\, du\, dv$

Area $= 2\pi \cdot \pi = 2\pi^2$



Exercises:

Green.

1. Compute $\int_C y^3\,dx - x^3\,dy$ where $C$ is the circle centered at the origin of radius 2
2. Determine whether $F = \langle e^z, y, xe^z \rangle$ is conservative, if it is, find its potential
3. Identify the surface with the given vector equation $\langle 2\sin u, 3\cos u, v\rangle, 0 \le v \le 2$

**13–18** Match the equations with the graphs labeled I–VI and give reasons for your answers. Determine which families of grid curves have $u$ constant and which have $v$ constant.

$\boxed{13.}$ ~~$\mathbf{r}(u, v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + v\,\mathbf{k}$~~

**14.** $\mathbf{r}(u, v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + \sin u\,\mathbf{k}, \quad -\pi \le u \le \pi$

**15.** $\mathbf{r}(u, v) = \sin v\,\mathbf{i} + \cos u \sin 2v\,\mathbf{j} + \sin u \sin 2v\,\mathbf{k}$

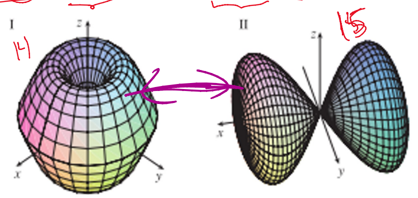
**16.** $x = (1 - u)(3 + \cos v)\cos 4\pi u,$
$y = (1 - u)(3 + \cos v)\sin 4\pi u,$
$z = 3u + (1 - u)\sin v$

$u = 1$, $x = 0$, $y = 0$, $z = 3$

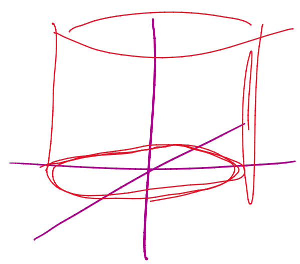
~~**17.** $x = \cos^3 u \cos^3 v, \quad y = \sin^3 u \cos^3 v,$~~ $\quad z = \sin^3 v$

~~**18.** $x = (1 - |u|)\cos v, \quad y = (1 - |u|)\sin v, \quad z = u$~~

I 14     II 15



III 17



14

4.

$\iint -3x^2 - 3y^2 \, dx\,dy = -3\iint r^2 r \, dr\,d\theta = -6\pi\left(\frac{2^4}{4}\right) = -24\pi$

y,

$F = xe^z + \frac{y^2}{2} + C$

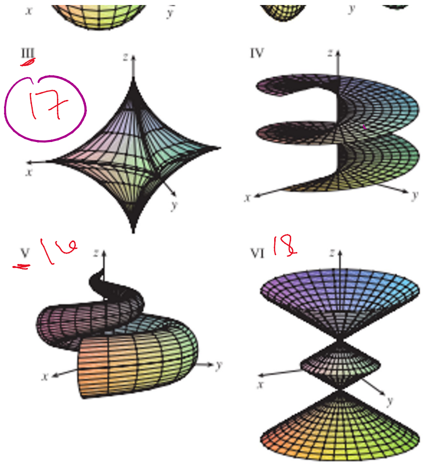
$\nabla \times F = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} e^z \\ y \\ xe^z \end{pmatrix} = \begin{pmatrix} 0 \\ e^z - e^z \\ 0 \end{pmatrix} = 0$

$F \overset{?}{=} (f_x, f_y, f_z)$

$f_{xy} = f_{yx} \qquad f_{xz} = f_{zx}$

if $f_{xy} \ne f_{yx} \Rightarrow F$ not conservative

4.




13

if $f'_{xy} \neq f'_{yx} \Rightarrow F$ not conservat.

$xe^z + g(y,z)$ $\quad$ $xe^z + \frac{y^2}{2} + g(z)$

$\int dx$ $\quad$ $\partial_y$ $\quad$ $\partial_y g = y \Rightarrow g = \frac{y^2}{2} + c$ $\quad$ $g'(z) = 0$

$e^z$ $\quad$ $\stackrel{?}{=} y$

$xe^z + g(y,z) = \frac{y^2}{2} + g_2(x,z) = xe^z + g_3(x,y)$

$e^z$ $\quad$ $y$ $\quad$ $xe^z$